The Mathematical Correctness of Differential Geometry
(A Refutation of Rodrigues and de Souza)

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Abstract

The refutation of Rodrigues and de Souza in this paper uses well-known methods to prove that they were incorrect mathematically in their attempt to rebut Evans unified field theory by rebutting standard differential geometry. These methods include a derivation of the standard tetrad postulate. The Evans unified field theory is a straightforward unification scheme based on standard differential geometry. Unification has been achieved through the realization that the electromagnetic field is spinning spacetime governed by the Maurer Cartan structure relations of differential geometry, from which follows the well-known tetrad postulate. Within a fundamental, universal or primordial voltage, the electromagnetic potential field is the fundamental tetrad field, the electromagnetic field tensor is the torsion form, and the Evans spin field observed in the inverse Faraday effect is described by the spin connection of differential geometry. The first Bianchi identity produces the four laws of classical electrodynamics unified with gravitation.

Keywords: Evans unified field theory; standard differential geometry.

The following is a refutation of the paper: W. A. Rodrigues Jr. and Q. A. G. de Souza, *An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields* [arxiv.org/pdf/math-ph/0411085v3].

Introduction

There are many textbooks and university courses available on standard differential geometry and the related subject of geometric algebra. A particularly clear example for the use of physicists is the graduate course given at Harvard, UC Santa Barbara and the University of Chicago by Carroll [1]. Any rebuttal of standard differential geometry would have to overturn the basic mathematical work of a hundred and seventy years or more, going back to Riemann and Clifford. It seems hardly necessary, therefore, for a physicist to have to defend standard textbook mathematics, but such a defense has been rendered necessary by a preprint posted by Rodrigues and de Souza [2], a preprint consisting of many pages of irrelevant polemic and meaningless, incomprehensible abstraction. In order to deal with such material, we need only cite standard differential geometry in a manner that is comprehensible to mathematicians, physicists and engineers.

The Evans unified field theory is based directly on standard, or textbook, differential geometry. A simple type of field unification has been shown through a series of twenty-five papers initiated circa March 2003 [4]. Evans realized that electromagnetism in general relativity is spinning spacetime and is governed and unified with gravitation by the Maurer Cartan structure equations of standard differential geometry. Within a fundamental, primordial or universal voltage $cA^{(0)}$, the electromagnetic potential is the fundamental tetrad one-form of differential geometry:

$$A_{\mu}^{a} = A^{(0)} q_{\mu}^{a} \ , \tag{1}$$

and the electromagnetic field is the torsion two-form defined by the first Maurer Cartan structure relation as the covariant exterior derivative of the tetrad:

$$F_{\mu\nu}^{a} = A^{(0)} T_{\mu\nu}^{a} = A^{(0)} \left(d \wedge q^{a} + \omega_{b}^{a} \wedge q^{b} \right)_{\mu\nu} = A^{(0)} (D \wedge q^{a})_{\mu\nu} \ . \tag{2}$$

Here $\omega_{\mu b}^{a}$ is the spin connection of standard differential geometry, $d \wedge$ is the standard exterior derivative, and $D \wedge$ is the standard covariant exterior derivative. The second Maurer Cartan structure equation defines the Riemann form [1, 5-7] as the covariant exterior derivative of the spin connection.

From this realization, the four laws of classical electrodynamics have been unified with classical gravitation [8] using the first Bianchi identity of standard differential geometry:

$$(D \wedge T^{a})_{\mu\nu} = \left(R_{b}^{a} \wedge q^{b} \right)_{\mu\nu} \ . \tag{3}$$

The second Bianchi identity:

$$(D \wedge R_{b}^{a})_{\mu\nu} = 0 \tag{4}$$

produces the Noether Theorem of the Evans unified field theory [4, 8] (also see UFT Paper 88, Section 3).

The Evans spin field, well observed experimentally [9] in the inverse Faraday effect, is described by the spin connection through the second term of Eq. (2).

The Evans field theory has been cross checked mathematically in many ways [4]. It has been shown to produce the equations of both classical and quantum physics in appropriate limits, as required by a unification scheme, and has also been checked in many ways [9] with experimental data.

Rodrigues and de Souza have attempted to “refute” the Evans theory by attacking the fundamental tetrad postulate of standard differential geometry:

$$D_{\nu} q_{\mu}^{a} = 0 \ , \tag{5}$$

in a long and meaningless polemic.

The tetrad postulate is at the root of differential geometry because it links the Maurer Cartan structure equations with Riemann geometry. Complete details of how this is done have been given in several appendices of [10]. These appendices start with the Maurer Cartan structure equations and Bianchi identities of standard differential geometry, and correctly produce from them the fundamental structure equations and Bianchi identities of standard Riemann geometry, equations of the early nineteenth century.

This, in itself, is a clear demonstration of the correctness of the tetrad postulate in its well-known Palatini variation [5-7], the most general form of the postulate used in the Evans unified field theory. Demonstrations of the correctness of the tetrad postulate have also been given in [10], in all detail.

Furthermore, details about the tetrad postulate can also be found in courses and textbooks on differential geometry such as [5], and in contemporary papers on gravitation theory, for example [6, 7]. In addition, the tetrad is even used in standard quantum field theory, for example, see [11]. Among the distinguished colleagues whose work is polemicized indirectly or directly by Rodrigues and de Souza are Atiyah, Singer, Wheeler, Witten, Green, and anyone who uses differential geometry and the tetrad postulate (many thousands of authors).

In fact, all that is actually needed to refute Rodrigues and de Souza is to cite the standard derivation of the tetrad postulate as given, for example, by Carroll [1].

If an abstraction does not reduce to conventional mathematics (in this case, differential geometry and Riemann geometry) then it is meaningless to science. The exceedingly convoluted abstraction of Rodrigues and de Souza is an unscientific contrivance, as this paper demonstrates through the use of simpler and much less abstract, but correct, mathematics.

The derivation of the standard tetrad postulate is given in the next section. Some of the basic concepts of differential geometry that are needed for the tetrad postulate are first cited from a physics course given by Bertschinger at M.I.T. in the Spring Semester of 2002 [5].

A Simple Proof of the Tetrad Postulate

A vector basis is orthonormal [1, 5] if its dot product is given by the Minkowski metric at any point X in a base manifold. It is always possible to choose an orthonormal basis at any point X in a given manifold, and there are infinitely many orthonormal bases at X related to each other by a Lorentz transform. This defines the tangent bundle at any point X. The orthonormal basis defines the four vector V^a, and the same vector in the base manifold (Evans spacetime [4]) is denoted V^μ. The tetrad q^a_μ is defined as the matrix connecting V^a and V^μ :

$$V^a = q^a_\mu V^\mu \quad . \tag{6}$$

The tetrad is a vector valued one-form [1] and is always defined by Eq. (6). Consequently, it is not an unconstrained covariant four vector for each index *a*, and its covariant derivative, therefore, cannot be defined solely by the Christoffel connection:

$$D_\nu q^a_\mu \neq \partial_\nu q^a_\mu + \Gamma^\lambda_{\nu\mu} q^a_\lambda \quad . \tag{7}$$

The covariant derivative of the tetrad must be defined both by the Christoffel connection and the spin connection through the tetrad postulate:

$$D_\mu q^a_\lambda = \partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda - \Gamma^\nu_{\mu\lambda} q^a_\nu = 0 \tag{8}$$

in its most general form (the Palatini variation – in which there is no constraint on the connection).

If basis elements e_a are defined in the tangent bundle spacetime, and basis elements e_μ are defined in the base manifold, then these are also related by the tetrad:

$$e_\mu = q^a_\mu e_a \ . \tag{9}$$

It is shown in this section that Eq. (8) is a direct consequence of Eqs. (6) and (9).

Consider the vector field in the tangent bundle:

$$X = X^a e_a \ , \tag{10}$$

and the same vector field in the base manifold:

$$X = X^\mu e_\mu \ . \tag{11}$$

The corresponding covariant derivatives are

$$DX = \left(D_\mu X^\nu\right) dx^\mu \otimes e_\nu \tag{12}$$

and

$$DX = \left(D_\mu X^a\right) dx^\mu \otimes e_a \ . \tag{13}$$

Therefore, the covariant derivative of the contravariant vector X^ν has been defined by the Christoffel connection:

$$D_\mu x^\nu = \partial_\mu x^\nu + \Gamma^\nu_{\mu\lambda} x^\lambda \ , \tag{14}$$

and the covariant derivative of the vector X^a has been defined by the spin connection:

$$D_\mu X^a = \partial_\mu X^a + \omega^a_{\mu b} X^b \ . \tag{15}$$

The basis elements are connected by the tetrad as follows:

$$e_a = q^\sigma_a e_\sigma \ . \tag{16}$$

Using the commutator rule we may develop Eq. (13) as follows:

$$\begin{aligned} DX &= \left(\partial_\mu (q^a_\nu X^\nu) + \omega^a_{\mu b} q^b_\lambda X^\lambda\right) dx^\mu \otimes (q^\sigma_a e_\sigma) \\ &= q^\sigma_a \left(q^a_\nu\, \partial_\mu X^\nu + X^\nu\, \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\lambda X^\lambda\right) dx^\mu \otimes e_\sigma \ . \end{aligned} \tag{17}$$

Now switch the dummy indices σ to ν and use

$$q_a^\nu q_\nu^a = 1 \tag{18}$$

to obtain

$$DX = \left(\partial_\mu X^\nu + q_a^\nu\, \partial_\mu q_\lambda^a X^\lambda + q_a^\nu q_\lambda^b \omega_{\mu b}^a X^\lambda\right) dx^\mu \otimes e_\nu \ \ . \tag{19}$$

Compare Eqs. (12) and (19) to obtain

$$\Gamma_{\mu\lambda}^\nu = q_a^\nu\, \partial_\mu q_\lambda^a + q_a^\nu q_\lambda^b \omega_{\mu b}^a \ \ , \tag{20}$$

and multiply both sides of Eq. (20) by q_ν^a to obtain the tetrad postulate:

$$q_\nu^a \Gamma_{\mu\lambda}^\nu = \partial_\mu q_\lambda^a + q_\lambda^b \omega_{\mu b}^a \ \ . \tag{21}$$

Q.E.D.

Refutation of Trivial Errors by Rodrigues and de Souza

The paper by Rodrigues and de Souza [2] contains characteristic trivial errors. In this section, we confine ourselves to a refutation of two of these, which is sufficient to show the nature of their analysis.

The first error is their assertion that

$$D^\mu\left(D_\mu q_\nu^a\right) = \partial^\mu\left(D_\mu q_\nu^a\right) = 0 \tag{22}$$

is “meaningless”.

Unfortunately for them, the expression (22) can be expanded to give

$$\partial^\mu\left(\partial_\mu q_\lambda^a + \omega_{\mu b}^a q_\lambda^b - \Gamma_{\mu\lambda}^\nu q_\nu^a\right) = 0 \ \ , \tag{23}$$

whereupon it is seen that the first operator on the left side is the standard d'Alembertian operator.

This is the first time that the d'Alembertian has been described as meaningless.

The second trivial error dealt with here is their assertion that the definition of the wedge product of tetrads by Evans is somehow “non-standard”.

It is not clear whether Rodrigues and de Souza go so far as to assert that the wedge product used by Evans is incorrect. If so, then the whole of standard differential geometry is incorrect, because Evans [4] uses precisely the same wedge product as Carroll [1] and all mainstream mathematicians. A reasonable conclusion might be that Rodrigues and de Souza are either not familiar with the standard wedge product, or they are not familiar with the work that they are attempting to criticize.

References

[1] S. P. Carroll, *Lecture Notes in General Relativity* (for a graduate course at Harvard, UC Santa Barbara and the University of Chicago) [arxiv.org/abs/gr-qc/9712019].

[2] W. A. Rodrigues Jr. and Q. A. G. de Souza, *An Ambiguous Statement Called 'Tetrad Postulate' and the Correct Field Equations Satisfied by the Tetrad Fields* [arxiv.org/pdf/math-ph/0411085v3].

[3] The Evans field theory is described in detail in the UFT Papers available on www.aias.us.

[4] M. W. Evans, *Generally Covariant Unified Field Theory: The Geometrization of Physics* (Springer 2005, van der Merwe Series). Also available as UFT Papers 1-25 at [3].

[5] E. Bertschinger, Physics 8.962 course at M.I.T, Spring 2002 (www.ocw.mit.edu).

[6] D. N. Vollick, *On the Dirac Field in the Palatini Form of 1/R Gravity*, arXiv, grgc/0409068 v1 (2004), and references therein to contemporary Phys. Rev. D articles that contain the tetrad postulate in its most general Palatini variation, as used universally in differential geometry (and also in the Evans field theory).

[7] E. E. Flanagan, Phys. Rev. Lett., 92, 071 101 (2004). This is an example of a contemporary paper using the tetrad postulate in its first variation, with symmetric gamma connection.

[8] M. W. Evans, *The Objective Laws of Classical Electrodynamics: The Effect of Gravitation on Electromagnetism* (UFT Paper 26 at [3]).

[9] M. W. Evans, ed., *Modem Non-Linear Optics*, a special topics issue in three parts of I. Prigogine and S. A. Rice (series eds.), *Advances in Chemical Physics* (Wiley Interscience, New York, 2001, Second Edition), Vols. 119(2) and 119(3).

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[11] L. H. Ryder, *Quantum Field Theory* (Cambridge, 1996, Second Edition).

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